# Tutorial on the 100th Anniversary of Cholesky's Algorithm

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**Abstract**

On December 2, 1910 Andre Louis Cholesky described his now famous algorithm [1]. He called it "On the numerical solution of systems of linear equations". He wrote it down on seven plus pages of legal sized paper. He did not publish it! Fourteen years later, in 1924, Commander Benoit, seven years after Cholesky's tragic death during World War I, made Cholesky's Algorithm known to the world.

It seems fitting that we honor Cholesky at PPAM 2011. One hundred years later his algorithm is still the object of intense and far-reaching research. This is especially true for PPAM participants. Therefore, we shall describe Polish contributions to this area of Dense Linear Algebra. We plan to honor A. L. Cholesky, 1875-1917 and astronomer Tadeusz Banachiewicz, 1882-1954. In 1938 Tadeusz published a square root method closely related to Choleksy's Algorithm. In 1925, he invented a theory of "cracovians" (a special kind of matrix algebra) which brought him international recognition. Cracovians introduced the idea of using $\mathbf{A^T}$, and multiplying the columns of $\mathbf{A^T}$ by the column vector $\mathbf{x}$ to form the matrix vector product $\mathbf{y} = \mathbf{Ax}$. Banachiewicz was a professor at the Jagiellonian University and the director of the Cracow Observatory.

This Tutorial will be divided into four Lectures of about one hour each. In Lecture one we cover a history of Gaussian Elimination [2]. Gauss is given credit for "inventing" Gaussian Elimination. However, his major contribution in dense linear algebra is his development of the method of Least Squares which leads to solving $Ax = b$ where $A$ is positive definite. Gauss made seminal contributions in the field of astronomy and land survey which were two major areas of applied mathematics during his lifetime. In Lecture two, we cover Gauss's specfic contributions in some detail [4]. What will be seen is that he developed a lot of modern day algorithmic matrix theory well before matrices were invented. Lecture three will be devoted to Cholesky's and Banaschiewicz's contributions to the problems of land survey and astronomy. Both Gauss and Cholesky were interested in geodetic land survey. In Lecture four we will cover a modern day solution of Cholesky's Algorithm that uses near minimal storage [3]; this is a major feature of Cholesky's algorithm over competing algorithms.

## 1 Lecture Schedule on Sunday September 11, 2011

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12:30 to 13:00 Coffee Break 1
13:00 to 14:00 Lecture 1: History of Gaussian Elimination
14:15 to 14:15 Questions and Discussions on Lecture 1
14:15 to 15:15 Lecture 2: Gauss's Specific Contrbutions
15:15 to 15:30 Questions and Discussions on Lecture 2
15:30 to 16:00 Coffee Break 2
16:00 to 17:00 Lecture 3: Cholesky's and Banachiewicz's Contributions
17:00 to 17:15 Questions and Discussions on Lecture 3
17:15 to 18:15 Lecture 4: A Modern Day Minimal Storage Cholesky Algorithm
18:15 to 18:30 Questions and Discussions on Lecture 4
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# References


[1] Brezinski, C. The life and work of André Cholesky. *Numerical Algorithms* Vol. 43 No. 3, pp. 279-288, 2006.

[2] J. F. Grcar. Mathematicians of Gaussian Elimination. *Notices of the AMS* Vol. 58, No. 6, pp. 782-792, June-July, 2011.

[3] F. G. Gustavson, J. Waśniewski, J. J. Dongarra, J. Herrero, J. Langou. Level-3 Cholesky Factorization Routines as Part of Many Cholesky Algorithms. *Transactions on Mathematical Software*, in $2^{nd}$ revision. also, LAWN No. 249, 23 pages, June, 2011.

[4] G. W. Stewart. Gauss, statistics, and Gaussian Elinination. Institute for Advanced Computer Studies TR-94-78 Dept. of Computer Science TR-3307, U. of Maryland, 8-94 13 pages.